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Response Surface Methodology-Based Optimisation of Finite Element Model Updating for Disc Brake Dynamic Behaviour: A Review

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ABSTRACT

Accurate modelling and prediction of disc brake dynamic behaviour are essential for addressing vibration and noise issues in automotive braking systems. Finite element model updating (FEMU) has been widely adopted to improve the correlation between numerical simulations and experimental modal data; however, conventional updating approaches are often limited by iterative trial-and-error procedures, high computational cost, and insufficient consideration of parameter interactions. In recent years, Response Surface Methodology (RSM) has emerged as a promising surrogate-based optimisation technique to enhance the efficiency and robustness of FEMU frameworks. This paper presents a comprehensive review and synthesis of RSM-based optimisation approaches integrated with FEMU for disc brake dynamic analysis. Key developments in surrogate modelling, design of experiments, and hybrid optimisation strategies are critically examined, with particular emphasis on their capability to manage high-dimensional parameter spaces and complex nonlinear phenomena such as frictional contact and thermo-mechanical coupling. The role of advanced experimental modal analysis techniques in improving model fidelity and supporting optimisation accuracy is also discussed. Based on the reviewed literature, current limitations and research gaps are identified, including challenges related to nonlinear behaviour modelling, experimental–numerical integration, and automation of optimisation workflows. Finally, future research directions are outlined, highlighting the need for comprehensive, adaptive, and multi-physics RSM–FEMU frameworks to support reliable, efficient, and sustainable disc brake design and analysis.

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INTRODUCTION

The braking system plays a critical role in ensuring vehicle safety and ride comfort by controlling speed and enabling stable stopping performance. Among the various braking components, the disc brake rotor is directly subjected to friction-induced forces during braking events, which may lead to vibration and noise-related issues. These dynamic responses are closely associated with the modal characteristics of the disc brake structure, such as natural frequencies and mode shapes. Consequently, accurate identification and prediction of disc brake dynamic behaviour are essential, particularly in the early design and development stages of automotive systems.

Finite element analysis (FEA) has been widely employed as a numerical tool to predict the dynamic characteristics of disc brake structures due to its flexibility and cost efficiency. Modal analysis using FEA enables engineers to estimate natural frequencies and mode shapes prior to physical testing. However, the accuracy of numerical predictions is highly dependent on the fidelity of the finite element (FE) model, which is often influenced by modelling simplifications, uncertainties in material properties, and assumptions regarding boundary conditions. As a result, discrepancies frequently arise when numerical results are compared with experimentally measured data obtained from experimental modal analysis (EMA).

To mitigate these discrepancies, finite element model updating (FEMU) techniques have been extensively applied to improve the correlation between numerical predictions and experimental measurements. FEMU aims to adjust selected model parameters, such as material properties or stiffness-related variables, in order to minimise the differences between predicted and measured dynamic responses. Sensitivity-based and iterative updating approaches are among the most commonly adopted methods, as they allow systematic parameter adjustment based on changes in modal characteristics. Despite their effectiveness, these approaches often rely on repeated trial-and-error procedures and may require a large number of iterations to achieve convergence, particularly when multiple updating parameters are involved. Table 1 is summarized the key insights of optimisation process including RSM from past researchers.

Recent studies have demonstrated increasing interest in integrating Response Surface Methodology (RSM) with FEMU to enhance prediction accuracy and computational efficiency in complex automotive structures, including disc brake systems. Surrogate-based optimisation frameworks developed using RSM enable accurate approximation of system behaviour while significantly reducing the computational cost associated with repeated finite element simulations. When combined with advanced optimisation strategies, such approaches facilitate systematic exploration of design trade-offs and provide a more structured updating process compared to conventional iterative methods [1].

Nevertheless, accurate modelling of disc brake dynamics remains challenging due to the presence of nonlinear frictional contact and thermo-mechanical coupling effects. These nonlinearities play a dominant role in vibration-related phenomena such as brake squeal and transient response behaviour, where simplified assumptions may result in poor predictive capability. Consequently, several studies emphasise the need for advanced modelling strategies and robust constraint handling to capture these complex interactions more realistically within numerical frameworks [2-4].

To manage the growing complexity associated with multiple interacting parameters, adaptive RSM strategies and advanced design of experiments (DOE) techniques, such as central composite and Box–Behnken designs, have been widely adopted. These approaches enable efficient sampling of high-dimensional parameter spaces while maintaining surrogate model accuracy with a reduced number of simulations runs [5-7]. Such capabilities are particularly valuable in FEMU applications, where several material and structural parameters simultaneously influence dynamic response.

Furthermore, hybrid optimisation frameworks incorporating multi-objective algorithms within RSM–FEMU formulations have been reported to provide robust trade-offs between competing performance objectives, such as stiffness, stress, and frequency alignment. These optimisation strategies contribute to improved design reliability and consistency; however, their effectiveness strongly depends on the quality of experimental data used for model updating [1,8].

High-resolution experimental modal analysis techniques are therefore essential to ensure reliable FE model updating. While advances in experimental modal testing have improved measurement accuracy and modal identification capability, their integration within RSM-based FEMU optimisation frameworks remains limited and requires further investigation [9-10]. This gap highlights the need for a structured RSM-driven optimisation approach that systematically links experimental modal data with finite element model updating to enhance prediction accuracy of disc brake dynamic behaviour. Accordingly, this study proposes a Response Surface Methodology-based optimisation framework for finite element model updating of a disc brake structure, aiming to improve model reliability and support robust automotive vibration analysis.

Table 1: Summary of Optimisation Methods Including RSM from Past Researchers

Theme	Key Insights	Supporting Citations
Integration of RSM with FEMU	RSM-based surrogate models, when integrated with FEMU and advanced optimization (e.g., MOPSO), enable efficient, accurate prediction of disc brake dynamic and thermo-mechanical behaviour, reducing computational cost and improving design trade-offs.	[1]
Nonlinear Friction & Thermo-Mechanical Coupling	Nonlinear frictional contact and thermo-mechanical coupling remain major challenges; advanced models and constraint handling are needed for accurate squeal and transient response prediction.	[2-4]
Adaptive RSM & High-Dimensional DOE	Adaptive RSM strategies and advanced DOE (e.g., CCD, Box–Behnken) help manage high-dimensional parameter spaces, reducing simulation runs while maintaining surrogate model accuracy.	[5-7]
Hybrid Optimization Algorithms	Multi-objective optimization (e.g., MOPSO) within RSM-FEMU frameworks yields robust trade-offs between conflicting objectives (e.g., stress vs. frequency), improving design reliability.	[1,8]
Experimental Modal Analysis	High-resolution experimental modal testing (e.g., SLDV, PolyMax) is essential for accurate FE	[9,10]

Theme	Key Insights	Supporting Citations
	model updating, but integration with RSM-based frameworks needs further development.	

THEORETICAL FRAMEWORKS

Fundamentals and Advances in Finite Element Model Updating for Disc Brakes

Finite Element Modelling of Disc Brakes: Techniques and Advances

Modern finite element (FE) modelling of brake systems has evolved toward fully coupled thermo-mechanical analyses to realistically capture in-service behaviour. Transient thermal simulations are commonly integrated with static structural analyses, often incorporating CFD-derived heat transfer coefficients to represent temperature-dependent cooling effects, which are essential for accurately predicting temperature distributions, deformation, and stress under realistic braking conditions [3,11]. In parallel, advanced friction formulations that account for pressure-, speed-, and temperature-dependent behaviour, together with wear laws such as Archard's law, are increasingly incorporated to improve the fidelity of frictional contact and wear predictions [1,13].

Beyond thermal and contact realism, FE models are widely applied to assess dynamic stability and support design optimization. Complex Eigenvalue Analysis (CEA) is commonly used to identify unstable modes associated with brake squeal, while dynamic transient analyses capture divergent vibration responses, with both approaches being highly sensitive to contact regime and friction modelling choices [12]. These modelling frameworks are now frequently coupled with response surface methodology (RSM) and multi-objective optimization algorithms, such as multi-objective particle swarm optimization (MOPSO), to balance competing objectives including stress and temperature reduction and natural frequency enhancement [1]. Additionally, three-dimensional thermo-mechanical FE models are employed to investigate instabilities such as hot spotting and thermal judder, identifying thermoelastic buckling as a key mechanism influencing braking performance and stability [14].

Experimental Modal Testing and Model Updating

Recent advances in experimental–numerical correlation have significantly improved the accuracy of finite element (FE) model updating through high-fidelity testing and optimization techniques. High-resolution modal testing methods, such as Scanning Laser Doppler Vibrometer (SLDV) combined with PolyMax modal parameter identification, enable detailed non-contact measurement of modal parameters, thereby enhancing the reliability of experimental data used for FE model calibration [9,15]. These experimental inputs are commonly integrated with Particle Swarm Optimization (PSO)–based updating and sensitivity analysis to refine mass and stiffness matrices, leading to improved identification of dynamic coefficients, particularly near critical operating speeds [16]. Furthermore, the incorporation of small geometric deviations obtained from optical 3D measurement systems has been shown to further enhance the correlation between numerical predictions and experimental results, resulting in more realistic and robust FE models [17].

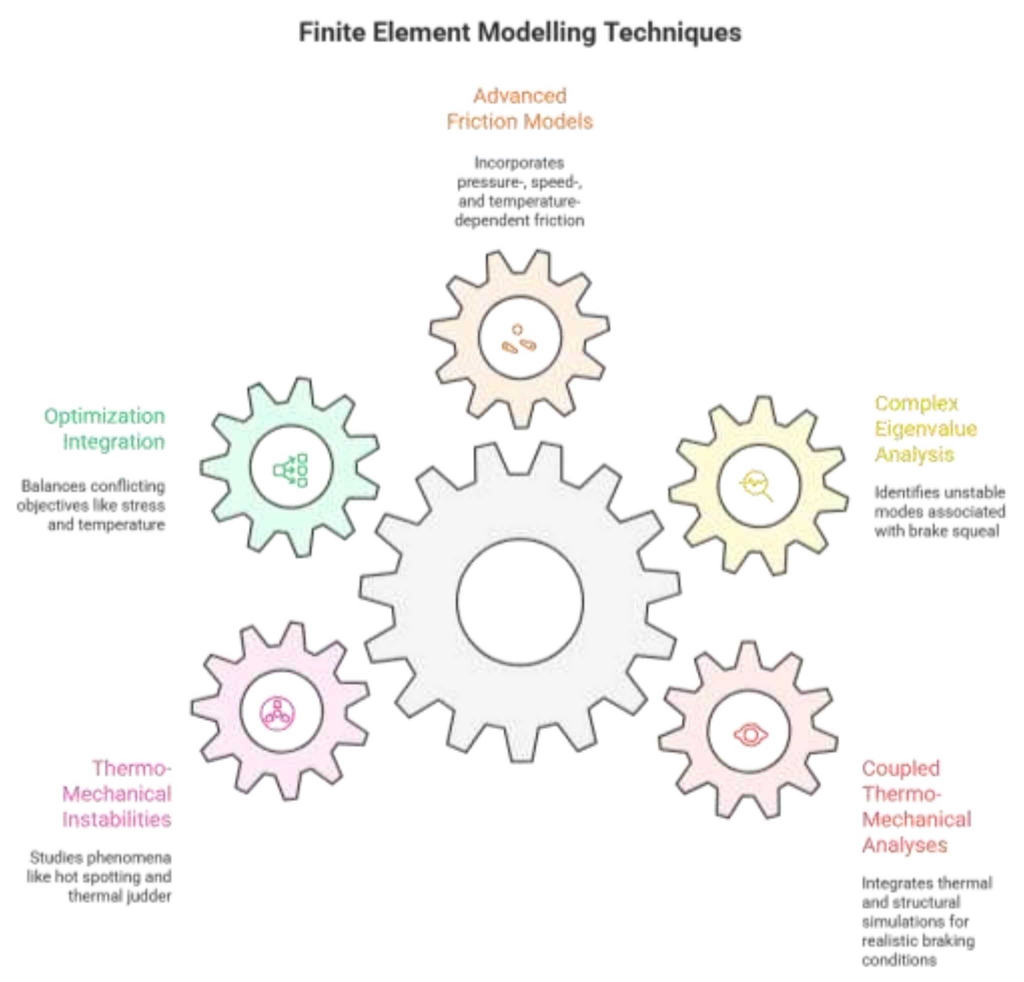

Figure 1: Summary of Finite Element Modelling Techniques.

Stochastic and Nonlinear Approaches in FEMU

Recent developments in finite element (FE) modelling and updating of disc brake systems increasingly focus on handling uncertainty, nonlinearities, and computational efficiency to improve predictive capability. Hybrid stochastic FEM approaches, including Variance-Based Sensitivity Analysis and the Fourier Sensitivity Analysis Test (FAST-FE), are used to efficiently propagate uncertainties and quantify parameter sensitivities, thereby enhancing the accuracy of brake squeal instability predictions [18]. In parallel, nonlinear constraint-handling techniques such as Sparse Identification of Nonlinear Dynamics (SINDy) and Lyapunov-based feedback control have been introduced to address friction-induced nonlinearities and complex system dynamics, leading to more robust and stable numerical models [19–21]. Overall, the evolution of FE modelling for disc brakes demonstrates a clear shift toward greater physical realism through multi-physics coupling and nonlinear formulations, combined with improved computational efficiency via surrogate modelling and advanced optimization methods, while rigorous experimental validation remains central to ensuring model credibility and reliability.

Principles and Applications of Response Surface Methodology in Engineering Optimization

Fundamental Principles of Response Surface Methodology

Response Surface Methodology (RSM) is a systematic optimization framework that combines experimental design and surrogate modelling to efficiently explore complex design spaces. The core steps of RSM include design of experiments (DOE), surrogate model construction—typically using polynomial regression—and optimization, with key stages such as factor screening, the path of steepest ascent, and iterative model refinement to improve accuracy [22–24]. Through surrogate modelling, RSM provides an efficient approximation of computationally expensive simulations, enabling rapid evaluation of design variables and identification of optimal parameter settings without repeated high-fidelity analyses [22]. In the context of finite element model updating (FEMU), RSM is widely applied to select influential FE model parameters, design structured experiments such as central composite designs, and construct response surfaces that guide parameter updating so that analytical predictions, particularly modal frequencies, closely match experimental measurements [25–26].

Experimental Designs and Adaptive Strategies in RSM

In Response Surface Methodology (RSM), various design of experiments (DOE) techniques are employed to efficiently sample the design space, with Central Composite Design (CCD), Box–Behnken Design (BBD), and orthogonal designs being among the most commonly used. CCD is often preferred due to its favourable balance between accuracy and computational efficiency, while BBD is particularly effective for capturing nonlinear interactions with a reduced number of experimental runs [27–28]. To further enhance efficiency, adaptive RSM strategies such as trust-region methods (e.g., STRONG) and adaptive sequential experimentation iteratively concentrate sampling in promising regions of the design space, thereby reducing the number of required simulations and improving convergence toward optimal solutions [5,29–30]. For high-dimensional finite element model updating (FEMU) problems, complexity and computational cost are managed through dimension-reduction techniques, including sensitivity analysis and parameter screening, as well as advanced surrogate modelling approaches such as polynomial chaos expansions [31–33]. Overall, RSM offers a flexible and efficient framework for surrogate modelling and optimization in engineering applications, with adaptive and high-dimensional strategies playing an increasingly important role in addressing large-scale, nonlinear problems.

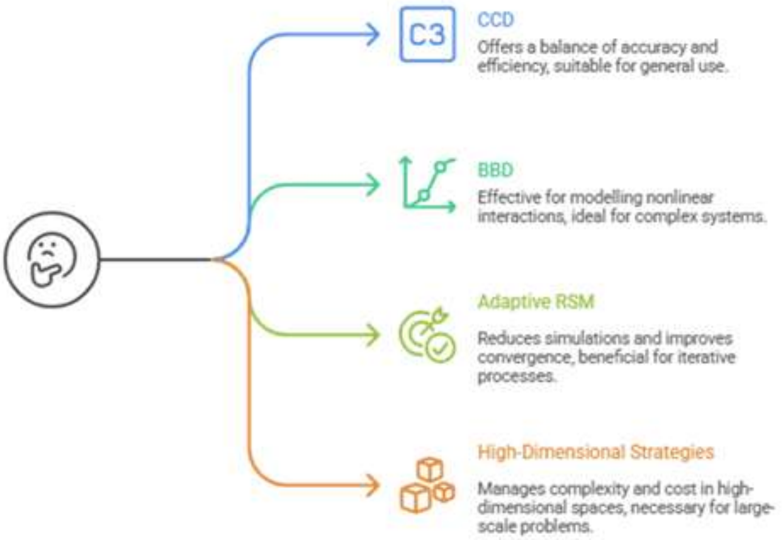

Figure 2: Experimental Designs and Adaptive Strategies in RSM

METHODS & DATA TRANSPARENCY

Integration of RSM with FEMU for Disc Brake Dynamics

Current Research and Gaps

The integration of finite element analysis (FEA), Response Surface Methodology (RSM), and advanced optimization algorithms such as multi-objective particle swarm optimization (MOPSO) has demonstrated significant success in enabling accurate and computationally efficient optimization of brake disc designs, with high predictive accuracy reported for stress, natural frequency, and temperature responses [1,34–35]. Experimental validation through finite element model updating (FEMU), supported by high-resolution modal testing techniques such as Scanning Laser Doppler Vibrometry (SLDV) and PolyMax identification, has been shown to improve the prediction of dynamic behaviour; however, its seamless integration within RSM-based optimization frameworks remains an area of ongoing development [36]. Despite these advances, accurately modelling nonlinear frictional contact and coupled thermal effects continues to pose significant challenges, with many studies resorting to simplified or decoupled representations due to high computational cost [13,37,38]. Consequently, a key research gap remains in the development of comprehensive frameworks that fully integrate RSM with FEMU for dynamic behaviour modelling, particularly to capture complex frictional and thermo-mechanical interactions in brake systems [1,34–35].

Multi-Objective Optimization: RSM, FEA, and MOPSO Integration

The application of multi-objective particle swarm optimization (MOPSO) to RSM-based surrogate models derived from finite element analysis (FEA) enables the generation of Pareto-optimal solutions, effectively balancing competing objectives such as minimizing stress and temperature while maximizing natural frequency [1,3]. The use of surrogate models significantly improves computational efficiency by reducing the need for repeated high-fidelity FEA simulations, allowing evolutionary algorithms to explore the design space more rapidly and effectively [1]. Validation of this integrated approach demonstrates high reliability, with discrepancies between predicted and simulated results typically remaining below 4%, confirming the accuracy of the methodology [1].

Nonlinear Frictional Contact Modelling in RSM-Based FEMU

Nonlinear joint behaviour in brake systems can be effectively captured using nonlinear spring elements with bilinear stiffness, which are updated through global optimization techniques and instantaneous dynamic response data to reflect realistic system dynamics [39]. Experimental insights, such as modal frequency–damping ratio maps obtained from impact hammer tests, inform the development of parametric joint models that accurately represent joint characteristics [40]. At the microscale, cross-scale contact models based on theoretical representations of microscopic contact mechanisms provide a comprehensive understanding of nonlinear stiffness and degradation, which is essential for precise dynamic predictions [41]. Furthermore, the integration of Response Surface Methodology (RSM) with sensitivity analysis and eigenvalue-based optimization enables targeted focus on the most influential variables, making multi-objective optimization tractable even in the presence of nonlinearities [42–43]. Overall, while the combination of RSM, FEMU, and advanced optimization algorithms is maturing, further research is required to fully capture nonlinear frictional and thermal effects and to streamline the integration of experimental data for automated model updating.

CRITICAL ANALYSIS OF FINDINGS

Real-World Implications

RSM-based finite element model updating (FEMU) frameworks offer powerful tools for the rapid and cost-effective optimization of disc brake designs, enabling engineers to balance competing objectives such as stress, temperature, and dynamic stability. This capability is particularly valuable in automotive and rail applications, where improvements in safety, noise reduction, and component lifespan are critical [1]. Enhanced model accuracy also supports predictive maintenance strategies, allowing for reduced downtime and increased system reliability. Experimental integration through high-resolution modal testing techniques, such as Scanning Laser Doppler Vibrometry (SLDV), provides essential data for model updating, although the incorporation of real-time or in-service measurements remains a challenge. Additionally, the use of surrogate modelling and adaptive design of experiments (DOE) strategies substantially reduces computational costs, making high-fidelity simulation and multi-objective optimization feasible even for complex, high-dimensional brake system models [5,30].

Limitations and Challenges

RSM-based finite element model updating (FEMU) frameworks provide notable advantages for brake system design and optimization, but several limitations remain. Accurately capturing nonlinear frictional contact and coupled thermo-mechanical effects is computationally demanding and often requires simplifications that can compromise model fidelity [2–4]. Handling high-dimensional parameter spaces demands advanced dimension-reduction and parameter-screening methods, which may unintentionally exclude influential variables [31,33]. Experimental validation is mostly confined to laboratory-scale studies, with full-scale, in-service verification still uncommon. Furthermore, automation for parametric modelling and response surface generation is in its early stages, restricting the speed and flexibility of design iteration [1,44]. In summary, while RSM-based FEMU frameworks offer substantial benefits, their effectiveness is constrained by challenges in modelling nonlinearities, managing complex high-dimensional systems, and incorporating real-time experimental data.

REAL-WORLD IMPLICATIONS

Improved disc brake modelling has a direct impact on multiple aspects of automotive and rail applications, including vehicle safety, noise reduction, and maintenance planning. RSM-based optimization facilitates the design of brake discs that achieve high performance while remaining manufacturable within cost and material constraints, supporting efficient production [1]. Accurate and updated finite element models also enhance diagnostics and health monitoring capabilities, enabling predictive maintenance strategies that are increasingly important in smart manufacturing and transportation systems. Furthermore, optimized brake designs can reduce material usage and extend component lifespan, contributing to broader sustainability goals by minimizing waste and resource consumption.

FUTURE RESEARCH DIRECTIONS

Coupling Thermomechanical and Dynamic Effects

Future research in brake system modelling will increasingly rely on advanced numerical methods to accurately and efficiently capture the coupled thermal, mechanical, and dynamic behaviours of disc brakes. Techniques such as transient thermal analysis, CFD–FEM coupling, and Eulerian-based multi-body approaches are essential for simulating these interactions with high fidelity [3,45–46]. Additionally, integrating wear and micro-thermal phenomena, particularly under varying brake pad pressures, is expected to be a key focus to enhance predictive accuracy and understand localized degradation mechanisms [47–48]. The development of real-time models for temperature distribution and heat transfer, leveraging finite element simulations and Response Surface Methodology (RSM), also holds promise for improving safety and performance predictions in practical applications [49].

Advancements in Nonlinear Constraint Handling and Automation

Future advancements in brake system modelling are likely to focus on improving robustness, efficiency, and adaptability through nonlinear and data-driven approaches. Nonlinear constraint-handling techniques that incorporate physical constraints alongside data-driven models, such as Sparse Identification of Nonlinear Dynamics (SINDy) or neural networks, can enhance the accuracy and stability of nonlinear model identification [19,50]. Automation through secondary development of finite element (FE) software—for parametric modelling and response surface generation—can further improve simulation efficiency and accuracy [1,44]. Moreover, hybrid surrogate modelling approaches that combine Response Surface Methodology (RSM) with machine learning techniques, including neural networks and SINDy, have the potential to reduce the number of high-fidelity simulation runs while improving adaptability to complex, nonlinear phenomena.

Multi-Physics and Wear Coupling in Future FEMU Frameworks

Future research in disc brake modelling should prioritize the development of comprehensive frameworks that integrate thermo-mechanical, frictional, and wear phenomena within RSM-based finite element model updating (FEMU), enabling holistic optimization of brake performance [47–48,51]. The continued advancement of multi-objective optimization algorithms, such as MOPSO and genetic algorithms (GA), within RSM-FEMU frameworks will facilitate robust trade-offs among conflicting design objectives, including stress, temperature, and dynamic stability [1,8]. Additionally, the real-time integration of experimental data streams into adaptive RSM frameworks has the potential to transform on-the-fly FEMU, enhancing brake system diagnostics and predictive maintenance capabilities. Overall, future efforts should focus on addressing gaps in nonlinear multi-physics modelling, automation, and real-time data integration to fully realize the potential of RSM-based FEMU for capturing and optimizing disc brake dynamic behaviour.

CONCLUSION

The integration of Response Surface Methodology with Finite Element Model Updating represents a significant advance in the modelling and optimization of disc brake dynamic behaviour. RSM-based surrogate models, when combined with advanced experimental modal analysis and multi-objective optimization algorithms, enable efficient, accurate prediction and optimization of thermo-mechanical and dynamic performance. However, challenges remain in fully capturing nonlinear frictional contact, thermo-mechanical coupling, and wear phenomena, as well as in automating and scaling these frameworks for real-time, in-service applications.

Emerging trends include the adoption of adaptive DOE strategies, hybrid surrogate models, and real-time data integration. Future research should prioritize the development of comprehensive, automated, and multi-physics frameworks that can support the next generation of high-performance, reliable, and sustainable disc brake systems [1, 25, 52].

Table 2: Comparative Analysis of RSM-Based FEMU Approaches for Disc Brake Dynamics

Approach	Strengths	Limitations	References
RSM + FEA + MOPSO	Efficient multi-objective optimization, high predictive accuracy, robust trade-offs	Limited by surrogate model fidelity, challenges in nonlinear/thermal coupling	[1]
Adaptive RSM (Trust-Region, Sequential)	Reduces simulation runs, improves convergence in high-dimensional spaces	May require complex implementation, risk of missing global optima	[29-30]
Nonlinear Frictional Contact Models	Captures key nonlinearities affecting squeal and dynamic behavior	Computationally intensive, often limited to simplified models	[4, 39]
Experimental Modal Updating (SLDV, PolyMax)	High-fidelity model updating, supports validation	Integration with RSM frameworks underdeveloped	[9-10]

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